

GATE 2022 Mathematics (MA)

Useful data

$A \setminus B$	$\{a \in A : a \notin B\}$
\mathbb{C}	Set of all complex numbers
$\mathbb{C}^{m \times n}$	Set of all matrices of order $m \times n$ with complex entries
$\mathbb{C}^\infty(\Omega)$	Collection of all infinitely differentiable functions on the open domain Ω
i	$\sqrt{-1}$
I	Identity matrix of appropriate order
$L^2(\mathbb{R})$	$:= L^2(\mathbb{R}, dx)$
$L^2[a, b]$	$:= L^2([a, b], dx)$
\mathbb{N}	Set of all positive integers
\mathbb{Q}	Set of all rational numbers
\mathbb{R}	Set of all real numbers
$\mathbb{R}^{m \times n}$	Set of all matrices of order $m \times n$ with real entries
\mathbb{S}^1	$\{(x_1, x_2) \in \mathbb{R}^2 : x_1^2 + x_2^2 = 1\}$
\mathbb{S}^2	$\{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1^2 + x_2^2 + x_3^2 = 1\}$
\mathbb{Z}	Set of all integers



GATE 2022 General Aptitude (GA)

Q.1 – Q.5 Carry ONE mark each.

Q.1	As you grow older, an injury to your _____ may take longer to _____ .
(A)	heel / heel
(B)	heal / heel
(C)	heal / heal
(D)	heel / heal



Q.2	<p>In a 500 m race, P and Q have speeds in the ratio of 3 : 4. Q starts the race when P has already covered 140 m.</p> <p>What is the distance between P and Q (in m) when P wins the race?</p>
(A)	20
(B)	40
(C)	60
(D)	140



Q.3	<p>Three bells P, Q, and R are rung periodically in a school. P is rung every 20 minutes; Q is rung every 30 minutes and R is rung every 50 minutes.</p> <p>If all the three bells are rung at 12:00 PM, when will the three bells ring together again the next time?</p>
(A)	5:00 PM
(B)	5:30 PM
(C)	6:00 PM
(D)	6:30 PM



<p>Q.4</p>	<p>Given below are two statements and four conclusions drawn based on the statements.</p> <p>Statement 1: Some bottles are cups.</p> <p>Statement 2: All cups are knives.</p> <p>Conclusion I: Some bottles are knives.</p> <p>Conclusion II: Some knives are cups.</p> <p>Conclusion III: All cups are bottles.</p> <p>Conclusion IV: All knives are cups.</p> <p>Which one of the following options can be logically inferred?</p>
<p>(A)</p>	<p>Only conclusion I and conclusion II are correct</p>
<p>(B)</p>	<p>Only conclusion II and conclusion III are correct</p>
<p>(C)</p>	<p>Only conclusion II and conclusion IV are correct</p>
<p>(D)</p>	<p>Only conclusion III and conclusion IV are correct</p>

<p>Q.5</p>	<p>The figure below shows the front and rear view of a disc, which is shaded with identical patterns. The disc is flipped once with respect to any one of the fixed axes 1-1, 2-2 or 3-3 chosen uniformly at random.</p> <p>What is the probability that the disc DOES NOT retain the same front and rear views after the flipping operation?</p> <div style="text-align: center;"> </div>
<p>(A) 0</p>	
<p>(B) $\frac{1}{3}$</p>	
<p>(C) $\frac{2}{3}$</p>	
<p>(D) 1</p>	



Q. 6 – Q. 10 Carry TWO marks each.

<p>Q.6</p>	<p>Altruism is the human concern for the wellbeing of others. Altruism has been shown to be motivated more by social bonding, familiarity and identification of belongingness to a group. The notion that altruism may be attributed to empathy or guilt has now been rejected.</p> <p>Which one of the following is the CORRECT logical inference based on the information in the above passage?</p>
<p>(A)</p>	<p>Humans engage in altruism due to guilt but not empathy</p>
<p>(B)</p>	<p>Humans engage in altruism due to empathy but not guilt</p>
<p>(C)</p>	<p>Humans engage in altruism due to group identification but not empathy</p>
<p>(D)</p>	<p>Humans engage in altruism due to empathy but not familiarity</p>



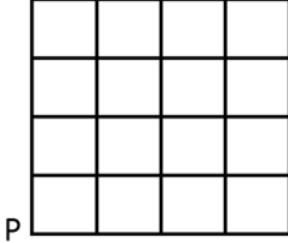
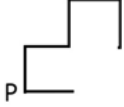

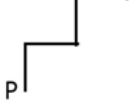

<p>Q.7</p>	<p>There are two identical dice with a single letter on each of the faces. The following six letters: Q, R, S, T, U, and V, one on each of the faces. Any of the six outcomes are equally likely.</p> <p>The two dice are thrown once independently at random.</p> <p>What is the probability that the outcomes on the dice were composed only of any combination of the following possible outcomes: Q, U and V?</p>
<p>(A)</p>	<p>$\frac{1}{4}$</p>
<p>(B)</p>	<p>$\frac{3}{4}$</p>
<p>(C)</p>	<p>$\frac{1}{6}$</p>
<p>(D)</p>	<p>$\frac{5}{36}$</p>



Q.8	<p>The price of an item is 10% cheaper in an online store S compared to the price at another online store M. Store S charges ₹ 150 for delivery. There are no delivery charges for orders from the store M. A person bought the item from the store S and saved ₹ 100.</p> <p>What is the price of the item at the online store S (in ₹) if there are no other charges than what is described above?</p>
(A)	2500
(B)	2250
(C)	1750
(D)	1500



<p>Q.9</p>	<p>The letters P, Q, R, S, T and U are to be placed one per vertex on a regular convex hexagon, but not necessarily in the same order.</p> <p>Consider the following statements:</p> <ul style="list-style-type: none"> • The line segment joining R and S is longer than the line segment joining P and Q. • The line segment joining R and S is perpendicular to the line segment joining P and Q. • The line segment joining R and U is parallel to the line segment joining T and Q. <p>Based on the above statements, which one of the following options is CORRECT?</p>
<p>(A)</p>	<p>The line segment joining R and T is parallel to the line segment joining Q and S</p>
<p>(B)</p>	<p>The line segment joining T and Q is parallel to the line joining P and U</p>
<p>(C)</p>	<p>The line segment joining R and P is perpendicular to the line segment joining U and Q</p>
<p>(D)</p>	<p>The line segment joining Q and S is perpendicular to the line segment joining R and P</p>

<p>Q.10</p>	<div style="text-align: center;">  </div> <p>An ant is at the bottom-left corner of a grid (point P) as shown above. It aims to move to the top-right corner of the grid. The ant moves only along the lines marked in the grid such that the current distance to the top-right corner strictly decreases.</p> <p>Which one of the following is a part of a possible trajectory of the ant during the movement?</p>
<p>(A)</p>	
<p>(B)</p>	
<p>(C)</p>	
<p>(D)</p>	

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Q.11 – Q.35 Carry ONE mark each.

<p>Q.11</p>	<p>Suppose that the characteristic equation of $M \in \mathbb{C}^{3 \times 3}$ is</p> $\lambda^3 + \alpha\lambda^2 + \beta\lambda - 1 = 0,$ <p>where $\alpha, \beta \in \mathbb{C}$ with $\alpha + \beta \neq 0$. Which of the following statements is TRUE?</p>
<p>(A)</p>	<p>$M(I - \beta M) = M^{-1}(M + \alpha I)$</p>
<p>(B)</p>	<p>$M(I + \beta M) = M^{-1}(M - \alpha I)$</p>
<p>(C)</p>	<p>$M^{-1}(M^{-1} + \beta I) = M - \alpha I$</p>
<p>(D)</p>	<p>$M^{-1}(M^{-1} - \beta I) = M + \alpha I$</p>



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<p>Q.12</p>	<p>Consider</p> <p>P: Let $M \in \mathbb{R}^{m \times n}$ with $m > n \geq 2$. If $\text{rank}(M) = n$, then the system of linear equations $Mx = 0$ has $x = 0$ as the only solution.</p> <p>Q: Let $E \in \mathbb{R}^{n \times n}, n \geq 2$ be a non-zero matrix such that $E^3 = 0$. Then $I + E^2$ is a singular matrix.</p> <p>Which of the following statements is TRUE?</p>
<p>(A)</p>	<p>Both P and Q are TRUE</p>
<p>(B)</p>	<p>Both P and Q are FALSE</p>
<p>(C)</p>	<p>P is TRUE and Q is FALSE</p>
<p>(D)</p>	<p>P is FALSE and Q is TRUE</p>

GATE 2022 Mathematics (MA)

Q.13	<p>Consider the real function of two real variables given by</p> $u(x, y) = e^{2x} [\sin 3x \cos 2y \cosh 3y - \cos 3x \sin 2y \sinh 3y].$ <p>Let $v(x, y)$ be the harmonic conjugate of $u(x, y)$ such that $v(0, 0) = 2$. Let $z = x + iy$ and $f(z) = u(x, y) + iv(x, y)$, then the value of $4 + 2if(i\pi)$ is</p>
(A)	$e^{3\pi} + e^{-3\pi}$
(B)	$e^{3\pi} - e^{-3\pi}$
(C)	$-e^{3\pi} + e^{-3\pi}$
(D)	$-e^{3\pi} - e^{-3\pi}$



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Q.14	<p>The value of the integral</p> $\int_C \frac{z^{100}}{z^{101} + 1} dz$ <p>where C is the circle of radius 2 centred at the origin taken in the anti-clockwise direction is</p>
(A)	$-2\pi i$
(B)	2π
(C)	0
(D)	$2\pi i$



GATE 2022 Mathematics (MA)

Q.15	Let X be a real normed linear space. Let $X_0 = \{x \in X : \ x\ = 1\}$. If X_0 contains two distinct points x and y and the line segment joining them, then, which of the following statements is TRUE?
(A)	$\ x + y\ = \ x\ + \ y\ $ and x, y are linearly independent
(B)	$\ x + y\ = \ x\ + \ y\ $ and x, y are linearly dependent
(C)	$\ x + y\ ^2 = \ x\ ^2 + \ y\ ^2$ and x, y are linearly independent
(D)	$\ x + y\ = 2\ x\ \ y\ $ and x, y are linearly dependent

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Q.16	<p>Let $\{e_k : k \in \mathbb{N}\}$ be an orthonormal basis for a Hilbert space H. Define $f_k = e_k + e_{k+1}, k \in \mathbb{N}$ and $g_j = \sum_{n=1}^j (-1)^{n+1} e_n, j \in \mathbb{N}$. Then $\sum_{k=1}^{\infty} \langle g_j, f_k \rangle ^2 =$</p>
(A)	0
(B)	j^2
(C)	$4j^2$
(D)	1



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Q.17	Consider \mathbb{R}^2 with the usual metric. Let $A = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}$ and $B = \{(x, y) \in \mathbb{R}^2 : (x-2)^2 + y^2 \leq 1\}$. Let $M = A \cup B$ and $N = \text{interior}(A) \cup \text{interior}(B)$. Then, which of the following statements is TRUE?
(A)	M and N are connected
(B)	Neither M nor N is connected
(C)	M is connected and N is not connected
(D)	M is not connected and N is connected



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Q.18	<p>The real sequence generated by the iterative scheme</p> $x_n = \frac{x_{n-1}}{2} + \frac{1}{x_{n-1}}, \quad n \geq 1$
(A)	converges to $\sqrt{2}$, for all $x_0 \in \mathbb{R} \setminus \{0\}$
(B)	converges to $\sqrt{2}$, whenever $x_0 > \sqrt{\frac{2}{3}}$
(C)	converges to $\sqrt{2}$, whenever $x_0 \in (-1, 1) \setminus \{0\}$
(D)	diverges for any $x_0 \neq 0$



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Q.19	The initial value problem $\frac{dy}{dx} = \cos(xy), \quad x \in \mathbb{R}, \quad y(0) = y_0,$ where y_0 is a real constant, has
(A)	a unique solution
(B)	exactly two solutions
(C)	infinitely many solutions
(D)	no solution



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Q.20	<p>If eigenfunctions corresponding to distinct eigenvalues λ of the Sturm-Liouville problem</p> $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} = \lambda y, \quad 0 < x < \pi,$ $y(0) = y(\pi) = 0$ <p>are orthogonal with respect to the weight function $w(x)$, then $w(x)$ is</p>
(A)	e^{-3x}
(B)	e^{-2x}
(C)	e^{2x}
(D)	e^{3x}



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Q.21	<p>The steady state solution for the heat equation</p> $\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} = 0, \quad 0 < x < 2, \quad t > 0,$ <p>with the initial condition $u(x, 0) = 0, \quad 0 < x < 2$ and the boundary conditions $u(0, t) = 1$ and $u(2, t) = 3, \quad t > 0$, at $x = 1$ is</p>
(A)	1
(B)	2
(C)	3
(D)	4



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Q.22	<p>Consider $([0, 1], T_1)$, where T_1 is the subspace topology induced by the Euclidean topology on \mathbb{R}, and let T_2 be <i>any</i> topology on $[0, 1]$. Consider the following statements:</p> <p>P : If T_1 is a proper subset of T_2, then $([0, 1], T_2)$ is not compact.</p> <p>Q : If T_2 is a proper subset of T_1, then $([0, 1], T_2)$ is not Hausdorff.</p> <p>Then</p>
(A)	P is TRUE and Q is FALSE
(B)	Both P and Q are TRUE
(C)	Both P and Q are FALSE
(D)	P is FALSE and Q is TRUE



GATE 2022 Mathematics (MA)

Q.23	Let $p : ([0, 1], T_1) \rightarrow (\{0, 1\}, T_2)$ be the quotient map, arising from the characteristic function on $[\frac{1}{2}, 1]$, where T_1 is the subspace topology induced by the Euclidean topology on \mathbb{R} . Which of the following statements is TRUE?
(A)	p is an open map but not a closed map
(B)	p is a closed map but not an open map
(C)	p is a closed map as well as an open map
(D)	p is neither an open map nor a closed map



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Q.24	<p>Set $X_n := \mathbb{R}$ for each $n \in \mathbb{N}$. Define $Y := \prod_{n \in \mathbb{N}} X_n$. Endow Y with the product topology, where the topology on each X_n is the Euclidean topology. Consider the set</p> $\Delta = \{(x, x, x, \dots) \mid x \in \mathbb{R}\}$ <p>with the subspace topology induced from Y. Which of the following statements is TRUE?</p>
(A)	Δ is open in Y
(B)	Δ is locally compact
(C)	Δ is dense in Y
(D)	Δ is disconnected

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Q.25	<p>Consider the linear system of equations $Ax = b$ with</p> $A = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 4 & 1 \\ 2 & 0 & 3 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}.$ <p>Which of the following statements are TRUE?</p>
(A)	<p>The Jacobi iterative matrix is $\begin{pmatrix} 0 & 1/4 & 1/3 \\ 1/3 & 0 & 1/3 \\ 2/3 & 0 & 0 \end{pmatrix}$</p>
(B)	<p>The Jacobi iterative method converges for any initial vector</p>
(C)	<p>The Gauss-Seidel iterative method converges for any initial vector</p>
(D)	<p>The spectral radius of the Jacobi iterative matrix is less than 1</p>



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Q.26	The number of non-isomorphic abelian groups of order $2^2 \cdot 3^3 \cdot 5^4$ is _____.



GATE 2022 Mathematics (MA)

Q.27	The number of subgroups of a cyclic group of order 12 is _____.



GATE 2022 Mathematics (MA)

Q.28	The radius of convergence of the series $\sum_{n \geq 0} 3^{n+1} z^{2n}, z \in \mathbb{C}$ is _____ (round off to TWO decimal places).

**GATE 2022 Mathematics (MA)**

Q.29	The number of zeros of the polynomial $2z^7 - 7z^5 + 2z^3 - z + 1$ in the unit disc $\{z \in \mathbb{C} : z < 1\}$ is _____.



GATE 2022 Mathematics (MA)

<p>Q.30</p>	<p>If $P(x)$ is a polynomial of degree 5 and</p> $\alpha = \sum_{i=0}^6 P(x_i) \left(\prod_{j=0, j \neq i}^6 (x_i - x_j)^{-1} \right),$ <p>where x_0, x_1, \dots, x_6 are distinct points in the interval $[2, 3]$, then the value of $\alpha^2 - \alpha + 1$ is _____.</p>



GATE 2022 Mathematics (MA)

Q.31	The maximum value of $f(x, y) = 49 - x^2 - y^2$ on the line $x + 3y = 10$ is _____.



GATE 2022 Mathematics (MA)

Q.32	If the function $f(x, y) = x^2 + xy + y^2 + \frac{1}{x} + \frac{1}{y}, x \neq 0, y \neq 0$ attains its local minimum value at the point (a, b) , then the value of $a^3 + b^3$ is _____ (round off to TWO decimal places).



GATE 2022 Mathematics (MA)

<p>Q.33</p>	<p>If the ordinary differential equation</p> $x^2 \frac{d^2\phi}{dx^2} + x \frac{d\phi}{dx} + x^2\phi = 0, \quad x > 0$ <p>has a solution of the form $\phi(x) = x^r \sum_{n=0}^{\infty} a_n x^n$, where a_n's are constants and $a_0 \neq 0$, then the value of $r^2 + 1$ is _____.</p>



GATE 2022 Mathematics (MA)

Q.34	The Bessel functions $J_\alpha(x)$, $x > 0$, $\alpha \in \mathbb{R}$ satisfy $J_{\alpha-1}(x) + J_{\alpha+1}(x) = \frac{2\alpha}{x} J_\alpha(x)$. Then, the value of $(\pi J_{\frac{3}{2}}(\pi))^2$ is _____.



GATE 2022 Mathematics (MA)

<p>Q.35</p>	<p>The partial differential equation</p> $7\frac{\partial^2 u}{\partial x^2} + 16\frac{\partial^2 u}{\partial x \partial y} + 4\frac{\partial^2 u}{\partial y^2} = 0$ <p>is transformed to</p> $A\frac{\partial^2 u}{\partial \xi^2} + B\frac{\partial^2 u}{\partial \xi \partial \eta} + C\frac{\partial^2 u}{\partial \eta^2} = 0,$ <p>using $\xi = y - 2x$ and $\eta = 7y - 2x$. Then, the value of $\frac{1}{12^3}(B^2 - 4AC)$ is _____.</p>



GATE 2022 Mathematics (MA)

Q.36 – Q.65 Carry TWO marks each.

Q.36	Let $\mathbb{R}[X]$ denote the ring of polynomials in X with real coefficients. Then, the quotient ring $\mathbb{R}[X]/(X^4 + 4)$ is
(A)	a field
(B)	an integral domain, but not a field
(C)	not an integral domain, but has 0 as the only nilpotent element
(D)	a ring which contains non-zero nilpotent elements



GATE 2022 Mathematics (MA)

<p>Q.37</p>	<p>Consider the following conditions on two proper non-zero ideals J_1 and J_2 of a non-zero commutative ring R.</p> <p>P: For any $r_1, r_2 \in R$, there exists a unique $r \in R$ such that $r - r_1 \in J_1$ and $r - r_2 \in J_2$.</p> <p>Q: $J_1 + J_2 = R$</p> <p>Then, which of the following statements is TRUE?</p>
<p>(A)</p>	<p>P implies Q but Q does not imply P</p>
<p>(B)</p>	<p>Q implies P but P does not imply Q</p>
<p>(C)</p>	<p>P implies Q and Q implies P</p>
<p>(D)</p>	<p>P does not imply Q and Q does not imply P</p>



GATE 2022 Mathematics (MA)

Q.38	Let $f : [-\pi, \pi] \rightarrow \mathbb{R}$ be a continuous function such that $f(x) > \frac{f(0)}{2}$, $ x < \delta$ for some δ satisfying $0 < \delta < \pi$. Define $P_{n,\delta}(x) = (1 + \cos x - \cos \delta)^n$, for $n = 1, 2, 3, \dots$. Then, which of the following statements is TRUE?
(A)	$\lim_{n \rightarrow \infty} \int_0^{2\delta} f(x) P_{n,\delta}(x) dx = 0$
(B)	$\lim_{n \rightarrow \infty} \int_{-2\delta}^0 f(x) P_{n,\delta}(x) dx = 0$
(C)	$\lim_{n \rightarrow \infty} \int_{-\delta}^{\delta} f(x) P_{n,\delta}(x) dx = 0$
(D)	$\lim_{n \rightarrow \infty} \int_{[-\pi, \pi] \setminus [-\delta, \delta]} f(x) P_{n,\delta}(x) dx = 0$



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<p>Q.39</p>	<p>P : Suppose that $\sum_{n=0}^{\infty} a_n x^n$ converges at $x = -3$ and diverges at $x = 6$. Then $\sum_{n=0}^{\infty} (-1)^n a_n$ converges.</p> <p>Q: The interval of convergence of the series $\sum_{n=2}^{\infty} \frac{(-1)^n x^n}{4^n \log_e n}$ is $[-4, 4]$.</p> <p>Which of the following statements is TRUE?</p>
<p>(A)</p>	<p>P is true and Q is true</p>
<p>(B)</p>	<p>P is false and Q is false</p>
<p>(C)</p>	<p>P is true and Q is false</p>
<p>(D)</p>	<p>P is false and Q is true</p>



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Q.40	<p>Let</p> $f_n(x) = \frac{x^2}{x^2 + (1 - nx)^2}, \quad x \in [0, 1], \quad n = 1, 2, 3, \dots$ <p>Then, which of the following statements is TRUE?</p>
(A)	$\{f_n\}$ is not equicontinuous on $[0, 1]$
(B)	$\{f_n\}$ is uniformly convergent on $[0, 1]$
(C)	$\{f_n\}$ is equicontinuous on $[0, 1]$
(D)	$\{f_n\}$ is uniformly bounded and has a subsequence converging uniformly on $[0, 1]$

**GATE 2022 Mathematics (MA)**

Q.41	Let (\mathbb{Q}, d) be the metric space with $d(x, y) = x - y $. Let $E = \{p \in \mathbb{Q} : 2 < p^2 < 3\}$. Then, the set E is
(A)	closed but not compact
(B)	not closed but compact
(C)	compact
(D)	neither closed nor compact



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Q.42	Let $T : L^2[-1, 1] \rightarrow L^2[-1, 1]$ be defined by $Tf = \tilde{f}$, where $\tilde{f}(x) = f(-x)$ almost everywhere. If M is the kernel of $I - T$, then the distance between the function $\phi(t) = e^t$ and M is
(A)	$\frac{1}{2}\sqrt{(e^2 - e^{-2} + 4)}$
(B)	$\frac{1}{2}\sqrt{(e^2 - e^{-2} - 2)}$
(C)	$\frac{1}{2}\sqrt{(e^2 - 4)}$
(D)	$\frac{1}{2}\sqrt{(e^2 - e^{-2} - 4)}$



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Q.43	Let X , Y and Z be Banach spaces. Suppose that $T : X \rightarrow Y$ is linear and $S : Y \rightarrow Z$ is linear, bounded and injective. In addition, if $S \circ T : X \rightarrow Z$ is bounded, then, which of the following statements is TRUE?
(A)	T is surjective
(B)	T is bounded but not continuous
(C)	T is bounded
(D)	T is not bounded

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<p>Q.44</p>	<p>The first derivative of a function $f \in C^\infty(-3, 3)$ is approximated by an interpolating polynomial of degree 2, using the data</p> $(-1, f(-1)), (0, f(0)) \text{ and } (2, f(2)).$ <p>It is found that</p> $f'(0) \approx -\frac{2}{3}f(-1) + \alpha f(0) + \beta f(2).$ <p>Then, the value of $\frac{1}{\alpha\beta}$ is</p>
<p>(A)</p>	<p>3</p>
<p>(B)</p>	<p>6</p>
<p>(C)</p>	<p>9</p>
<p>(D)</p>	<p>12</p>



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Q.45	The work done by the force $F = (x + y)\hat{i} - (x^2 + y^2)\hat{j}$, where \hat{i} and \hat{j} are unit vectors in \overrightarrow{OX} and \overrightarrow{OY} directions, respectively, along the upper half of the circle $x^2 + y^2 = 1$ from $(1, 0)$ to $(-1, 0)$ in the xy -plane is
(A)	$-\pi$
(B)	$-\frac{\pi}{2}$
(C)	$\frac{\pi}{2}$
(D)	π

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<p>Q.46</p>	<p>Let $u(x, t)$ be the solution of the wave equation</p> $\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0, \quad 0 < x < \pi, \quad t > 0,$ <p>with the initial conditions</p> $u(x, 0) = \sin x + \sin 2x + \sin 3x, \quad \frac{\partial u}{\partial t}(x, 0) = 0, \quad 0 < x < \pi$ <p>and the boundary conditions $u(0, t) = u(\pi, t) = 0, \quad t \geq 0$. Then, the value of $u\left(\frac{\pi}{2}, \pi\right)$ is</p>
<p>(A)</p>	<p>$-1/2$</p>
<p>(B)</p>	<p>0</p>
<p>(C)</p>	<p>$1/2$</p>
<p>(D)</p>	<p>1</p>



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Q.47	<p>Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a linear transformation defined by</p> $T((1, 2)) = (1, 0) \quad \text{and} \quad T((2, 1)) = (1, 1).$ <p>For $p, q \in \mathbb{R}$, let $T^{-1}((p, q)) = (x, y)$.</p> <p>Which of the following statements is TRUE?</p>
(A)	$x = p - q; y = 2p - q$
(B)	$x = p + q; y = 2p - q$
(C)	$x = p + q; y = 2p + q$
(D)	$x = p - q; y = 2p + q$

GATE 2022 Mathematics (MA)

<p>Q.48</p>	<p>Let $y = (\alpha, -1)^T$, $\alpha \in \mathbb{R}$ be a feasible solution for the dual problem of the linear programming problem</p> $\begin{aligned} &\text{Maximize: } 5x_1 + 12x_2 \\ &\text{subject to: } x_1 + 2x_2 + x_3 \leq 10 \\ &\quad \quad \quad 2x_1 - x_2 + 3x_3 = 8 \\ &\quad \quad \quad x_1, x_2, x_3 \geq 0. \end{aligned}$ <p>Which of the following statements is TRUE?</p>
<p>(A)</p>	<p>$\alpha < 3$</p>
<p>(B)</p>	<p>$3 \leq \alpha < 5.5$</p>
<p>(C)</p>	<p>$5.5 \leq \alpha < 7$</p>
<p>(D)</p>	<p>$\alpha \geq 7$</p>



GATE 2022 Mathematics (MA)

Q.49	Let K denote the subset of \mathbb{C} consisting of elements algebraic over \mathbb{Q} . Then, which of the following statements are TRUE?
(A)	No element of $\mathbb{C} \setminus K$ is algebraic over \mathbb{Q}
(B)	K is an algebraically closed field
(C)	For any bijective ring homomorphism $f : \mathbb{C} \rightarrow \mathbb{C}$, we have $f(K) = K$
(D)	There is no bijection between K and \mathbb{Q}

GATE 2022 Mathematics (MA)

Q.50	Let T be a Möbius transformation such that $T(0) = \alpha, T(\alpha) = 0$ and $T(\infty) = -\alpha$, where $\alpha = (-1 + i)/\sqrt{2}$. Let L denote the straight line passing through the origin with slope -1 , and let C denote the circle of unit radius centred at the origin. Then, which of the following statements are TRUE?
(A)	T maps L to a straight line
(B)	T maps L to a circle
(C)	T^{-1} maps C to a straight line
(D)	T^{-1} maps C to a circle



GATE 2022 Mathematics (MA)

Q.51	Let $a > 0$. Define $D_a : L^2(\mathbb{R}) \rightarrow L^2(\mathbb{R})$ by $(D_a f)(x) = \frac{1}{\sqrt{a}} f\left(\frac{x}{a}\right)$, almost everywhere, for $f \in L^2(\mathbb{R})$. Then, which of the following statements are TRUE?
(A)	D_a is a linear isometry
(B)	D_a is a bijection
(C)	$D_a \circ D_b = D_{a+b}$, $b > 0$
(D)	D_a is bounded from below



GATE 2022 Mathematics (MA)

Q.52	Let $\{\phi_0, \phi_1, \phi_2, \dots\}$ be an orthonormal set in $L^2[-1, 1]$ such that $\phi_n = C_n P_n$, where C_n is a constant and P_n is the Legendre polynomial of degree n , for each $n \in \mathbb{N} \cup \{0\}$. Then, which of the following statements are TRUE?
(A)	$\phi_6(1) = 1$
(B)	$\phi_7(-1) = 1$
(C)	$\phi_7(1) = \sqrt{\frac{15}{2}}$
(D)	$\phi_6(-1) = \sqrt{\frac{13}{2}}$



GATE 2022 Mathematics (MA)

Q.53	Let $X = (\mathbb{R}, T)$, where T is the smallest topology on \mathbb{R} in which all the singleton sets are closed. Then, which of the following statements are TRUE?
(A)	$[0, 1]$ is compact in X
(B)	X is not first countable
(C)	X is second countable
(D)	X is first countable



GATE 2022 Mathematics (MA)

Q.54	<p>Consider (\mathbb{Z}, T), where T is the topology generated by sets of the form</p> $A_{m,n} = \{m + nk \mid k \in \mathbb{Z}\},$ <p>for $m, n \in \mathbb{Z}$ and $n \neq 0$. Then, which of the following statements are TRUE?</p>
(A)	(\mathbb{Z}, T) is connected
(B)	Each $A_{m,n}$ is a closed subset of (\mathbb{Z}, T)
(C)	(\mathbb{Z}, T) is Hausdorff
(D)	(\mathbb{Z}, T) is metrizable

GATE 2022 Mathematics (MA)

<p>Q.55</p>	<p>Let $A \in \mathbb{R}^{m \times n}$, $c \in \mathbb{R}^n$ and $b \in \mathbb{R}^m$. Consider the linear programming primal problem</p> $\begin{aligned} &\text{Minimize: } c^T x \\ &\text{subject to: } Ax = b \\ &\quad \quad \quad x \geq 0. \end{aligned}$ <p>Let x^0 and y^0 be feasible solutions of the primal and its dual, respectively. Which of the following statements are TRUE?</p>
<p>(A)</p>	<p>$c^T x^0 \geq b^T y^0$</p>
<p>(B)</p>	<p>$c^T x^0 = b^T y^0$</p>
<p>(C)</p>	<p>If $c^T x^0 = b^T y^0$, then x^0 is optimal for the primal</p>
<p>(D)</p>	<p>If $c^T x^0 = b^T y^0$, then y^0 is optimal for the dual</p>

GATE 2022 Mathematics (MA)

<p>Q.56</p>	<p>Consider \mathbb{R}^3 as a vector space with the usual operations of vector addition and scalar multiplication. Let $x \in \mathbb{R}^3$ be denoted by $x = (x_1, x_2, x_3)$. Define subspaces W_1 and W_2 by</p> $W_1 := \{x \in \mathbb{R}^3 : x_1 + 2x_2 - x_3 = 0\}$ <p>and</p> $W_2 := \{x \in \mathbb{R}^3 : 2x_1 + 3x_3 = 0\}.$ <p>Let $\dim(U)$ denote the dimension of the subspace U.</p> <p>Which of the following statements are TRUE?</p>
<p>(A)</p>	<p>$\dim(W_1) = \dim(W_2)$</p>
<p>(B)</p>	<p>$\dim(W_1) + \dim(W_2) - \dim(\mathbb{R}^3) = 1$</p>
<p>(C)</p>	<p>$\dim(W_1 + W_2) = 2$</p>
<p>(D)</p>	<p>$\dim(W_1 \cap W_2) = 1$</p>



GATE 2022 Mathematics (MA)

<p>Q.57</p>	<p>Three companies C_1, C_2 and C_3 submit bids for three jobs J_1, J_2 and J_3. The costs involved per unit are given in the table below:</p> <table style="margin-left: auto; margin-right: auto;"> <tr> <td></td> <td>J_1</td> <td>J_2</td> <td>J_3</td> </tr> <tr> <td>C_1</td> <td>10</td> <td>12</td> <td>8</td> </tr> <tr> <td>C_2</td> <td>9</td> <td>15</td> <td>10</td> </tr> <tr> <td>C_3</td> <td>15</td> <td>10</td> <td>9</td> </tr> </table> <p>Then, the cost of the optimal assignment is _____.</p>		J_1	J_2	J_3	C_1	10	12	8	C_2	9	15	10	C_3	15	10	9
	J_1	J_2	J_3														
C_1	10	12	8														
C_2	9	15	10														
C_3	15	10	9														



GATE 2022 Mathematics (MA)

<p>Q.58</p>	<p>The initial value problem $\frac{dy}{dx} = f(x, y)$, $y(x_0) = y_0$ is solved by using the following second order Runge-Kutta method:</p> $K_1 = hf(x_i, y_i)$ $K_2 = hf(x_i + \alpha h, y_i + \beta K_1)$ $y_{i+1} = y_i + \frac{1}{4}(K_1 + 3K_2), \quad i \geq 0,$ <p>where h is the uniform step length between the points x_0, x_1, \dots, x_n and $y_i = y(x_i)$. The value of the product $\alpha\beta$ is _____ (round off to TWO decimal places).</p>



GATE 2022 Mathematics (MA)

Q.59	The surface area of the paraboloid $z = x^2 + y^2$ between the planes $z = 0$ and $z = 1$ is _____ (round off to ONE decimal place).



GATE 2022 Mathematics (MA)

Q.60	The rate of change of $f(x, y, z) = x + x \cos z - y \sin z + y$ at P_0 in the direction from $P_0(2, -1, 0)$ to $P_1(0, 1, 2)$ is _____.



GATE 2022 Mathematics (MA)

<p>Q.61</p>	<p>If the Laplace equation</p> $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 1 < x < 2, \quad 1 < y < 2$ <p>with the boundary conditions</p> $\frac{\partial u}{\partial x}(1, y) = y, \quad \frac{\partial u}{\partial x}(2, y) = 5, \quad 1 < y < 2$ <p>and</p> $\frac{\partial u}{\partial y}(x, 1) = \frac{\alpha x^2}{7}, \quad \frac{\partial u}{\partial y}(x, 2) = x, \quad 1 < x < 2$ <p>has a solution, then the constant α is _____.</p>

**GATE 2022 Mathematics (MA)**

Q.62	Let $u(x, y)$ be the solution of the first order partial differential equation $x \frac{\partial u}{\partial x} + (x^2 + y) \frac{\partial u}{\partial y} = u, \text{ for all } x, y \in \mathbb{R}$ satisfying $u(2, y) = y - 4, y \in \mathbb{R}$. Then, the value of $u(1, 2)$ is _____.



GATE 2022 Mathematics (MA)

<p>Q.63</p>	<p>The optimal value for the linear programming problem</p> $\begin{aligned} \text{Maximize: } & 6x_1 + 5x_2 \\ \text{subject to: } & 3x_1 + 2x_2 \leq 12 \\ & -x_1 + x_2 \leq 1 \\ & x_1, x_2 \geq 0 \end{aligned}$ <p>is _____.</p>



GATE 2022 Mathematics (MA)

Q.64	<p>A certain product is manufactured by plants P_1, P_2 and P_3 whose capacities are 15, 25 and 10 units, respectively. The product is shipped to markets M_1, M_2, M_3 and M_4, whose requirements are 10, 10, 10 and 20, respectively. The transportation costs per unit are given in the table below.</p> <table style="margin-left: auto; margin-right: auto; border-collapse: collapse;"> <tr> <td style="border: none;"></td> <td style="border: 1px solid black; padding: 2px;">M_1</td> <td style="border: 1px solid black; padding: 2px;">M_2</td> <td style="border: 1px solid black; padding: 2px;">M_3</td> <td style="border: 1px solid black; padding: 2px;">M_4</td> <td style="border: none;"></td> </tr> <tr> <td style="border: 1px solid black; padding: 2px;">P_1</td> <td style="border: 1px solid black; padding: 2px;">1</td> <td style="border: 1px solid black; padding: 2px;">3</td> <td style="border: 1px solid black; padding: 2px;">1</td> <td style="border: 1px solid black; padding: 2px;">3</td> <td style="border: 1px solid black; padding: 2px;">15</td> </tr> <tr> <td style="border: 1px solid black; padding: 2px;">P_2</td> <td style="border: 1px solid black; padding: 2px;">2</td> <td style="border: 1px solid black; padding: 2px;">2</td> <td style="border: 1px solid black; padding: 2px;">4</td> <td style="border: 1px solid black; padding: 2px;">1</td> <td style="border: 1px solid black; padding: 2px;">25</td> </tr> <tr> <td style="border: 1px solid black; padding: 2px;">P_3</td> <td style="border: 1px solid black; padding: 2px;">2</td> <td style="border: 1px solid black; padding: 2px;">1</td> <td style="border: 1px solid black; padding: 2px;">1</td> <td style="border: 1px solid black; padding: 2px;">2</td> <td style="border: 1px solid black; padding: 2px;">10</td> </tr> <tr> <td style="border: none;"></td> <td style="border: 1px solid black; padding: 2px;">10</td> <td style="border: 1px solid black; padding: 2px;">10</td> <td style="border: 1px solid black; padding: 2px;">10</td> <td style="border: 1px solid black; padding: 2px;">20</td> <td style="border: none;"></td> </tr> </table> <p>Then the cost corresponding to the starting basic solution by the Northwest-corner method is _____.</p>		M_1	M_2	M_3	M_4		P_1	1	3	1	3	15	P_2	2	2	4	1	25	P_3	2	1	1	2	10		10	10	10	20	
	M_1	M_2	M_3	M_4																											
P_1	1	3	1	3	15																										
P_2	2	2	4	1	25																										
P_3	2	1	1	2	10																										
	10	10	10	20																											



GATE 2022 Mathematics (MA)

Q.65	Let M be a 3×3 real matrix such that $M^2 = 2M + 3I$. If the determinant of M is -9 , then the trace of M equals _____.